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First-passage times and survival probabilities for particles moving in a field of random correlated forces

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Generalized diffusion equations for the density of a particle moving in one dimension under the influence of Gaussian noise, with Ornstein-Uhlenbeck correlations, are used to study first-passage times and survival probabilities in the presence of static traps. These diffusion equations have been derived for times that are either short or large compared to the correlation time τ and are used, in particular, near $\tau=0$ (limit of quasiperfect dynamic randomness) and near $\tau=\infty$ (limit of quasistatic randomness). The mean first-passage times scale with distance and with model parameters in the same way as do superdiffusion times derived from mean-square displacements. The long-time survival probability decays exponentially in the $\tau\rightarrow 0$ case and decays as a shrunk exponential, with an exponent $t^{4/3}$, for quasistatic forces. The short-time behavior of the survival probability, as well as the finite-τ corrections near $\tau=0$ and near $\tau=\infty$, are also analyzed.

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I. INTRODUCTION

In a recent paper [1] we have derived probability distributions for the displacement and for the velocity of a particle moving under the influence of a Gaussian random force which is exponentially correlated in time, with a correlation time τ (Ornstein-Uhlenbeck noise). The instantaneous position of the particle is a second-order stochastic process governed by the equation of motion

$$\ddot{x}=f(t) \ . \tag{1}$$

This equation can be viewed as a low-damping limit of the Langevin equation, $\ddot{x}+\gamma\dot{x}=f(t)$. For large γ the Langevin equation describes a "diffusion-limited" regime since the motion becomes then rapidly diffusive, while for $\gamma=0$ it describes a superdiffusive regime which has been called "random-force dominated" in a recent application to reaction kinetics in low-damping systems [2].

In this paper we apply our earlier results for displacement distributions [1] in the random-force-dominated regime ($\gamma=0$) to the study of two quantities which are of considerable practical interest and hence have been the object of many theoretical analyses. These quantities are the first-passage-time probability and the associated mean and mean-squared first-passage times, on the one hand, and the survival probability of the random process in a random medium with localized static traps, on the other hand. As in Ref. [1], we shall distinguish between two limiting time domains where the above quantities show qualitatively different behavior: a short-time domain including short-time intervals compared to the correlation time τ of fluctuations of the random force, and a long-time domain, $t\gg\tau$. For $t\gg\tau$ correlation effects are small and the randomness is predominantly dynamical for $\tau\rightarrow 0$ while for $t\ll\tau$ the random force acts, in first approximation, as a static force when $\tau\rightarrow\infty$. The frequently considered case of white-noise randomness corresponds to $\tau=0$.

The probability of a first-passage time t is the probability that the particle first leaves a given domain at time t. First-passage time distributions have been studied extensively for normal diffusion (Wiener-Einstein process) [3] and for various first-order additive or multiplicative stochastic processes, as well as for lattice random walks. A selection of review articles and books in the physics literature, which discuss first-passage time problems and their applications to topics as diverse as diffusion-controlled reactions, dissociation of diatomic molecules, diffusive escape of chemical reactants across potential barriers, meteorological and geophysical phenomena, fatigue problems in materials, stability problems in mechanical structures, etc., is listed under Refs. [4]–[9]. To our knowledge the first passage-time problem for the second-order process (1) with an Ornstein-Uhlenbeck noise term has not been discussed before.

Consider now the problem of random motion in a medium with traps. In the simplest case one has randomly moving A particles reacting instantly on contact with immobile B particles, thereby producing C particles which are inert and also immobile. The study of this type of reaction kinetics, where the B particles act as absorbing traps, finds applications, e.g., in problems of exciton trapping in molecular crystals, diffusion of vacancies, or interstitial in imperfect crystals, and, obviously, in various types of chemical and photosynthetic reactions. One is primarily interested in finding the survival probability, i.e., the probability for an A particle to be still in a trap-free region, without having been absorbed, at time t. The survival probability has been found exactly in the case of diffusion along a line with a low concentration of randomly distributed traps, by Balagurov and Vaks [10] and by Donsker and Varadhan [11]. The subject has been lucidly reviewed by Haus and Kehr [9]. An interesting variant of this type of diffusion-controlled reaction is the diffusion in a random medium with nucleation centers instead of trapping centers [12]. The nucleation centers lead to proliferation, rather than to the removal, of diffusing particles on contact with these centers. Moreover, while the trapping problem is closely related to the quantum problem of the density of states of an electron in the potential of random impurities [10,9], the probability density for a diffusing particle in a proliferating medium follows directly from the form of the density of states in the random electron problem [13].

On the other hand, the survival probability for the case of particles moving solely under the influence of the random force has been discussed qualitatively by Araujo *et al.* [2] for white-noise autocorrelations ($\tau=0$). These authors also discussed other aspects of the reaction kinetics for this case and presented numerical simulations. Here we give a more detailed analytic treatment, which generalizes the work of Balagurov and Vaks [10] for particles moving by normal diffusion. Our analysis shows that the result of Araujo *et al.* for the survival probability in one dimension is in error. Considering the effect of temporal correlations of the random force, we study, in particular, the asymptotic form of the survival probability for $\tau \to \infty$, i.e., for a statically correlated random force. After recalling the necessary results of Ref. [1] for the domains $t \gg \tau$ and $t \ll \tau$ at the beginning of Sec. II, we proceed in Sec. II A to discuss the first-passage time distribution and, more specifically, the mean and the mean-square first-passage times. In Sec. II B we then derive a general expression for the survival probability, which we analyze in detail for short and for long times.

II. FIRST-PASSAGE TIME PROBLEMS UNDER RANDOM FORCES

The density distribution for a particle whose dynamical evolution is governed by Eq. (1), with a Gaussian noise $f(t)$ which is correlated over a time τ, that is

$$\langle f(t)f(t')\rangle = f_0^2 h(t-t') , \quad \langle f(t)\rangle = 0 , \tag{2}$$

$$h(t) = (2\tau)^{-1}\exp(-t/\tau) , \tag{3}$$

has been found to be Gaussian both for short and for long correlation times [1] or, equivalently for any correlation time in the limits $t \gg \tau$ and $t \ll \tau$. This distribution was discussed in Ref. [1] starting from the Fokker-Planck equation for the joint probability distribution of the position and the velocity. However, the Gaussian form of the density distribution follows directly from the Gaussian nature of the random force, for any form of autocorrelation. Indeed Eq. (1) yields, by quadrature,

$$x(t) = \int_0^t dt' \int_0^{t'} dt'' f(t'') , \tag{1'}$$

which allows us to find the moments $\langle x^m(t)\rangle$ of the density distribution. With $\langle f(t)\rangle = 0$ the odd moments vanish and the even Gaussian moments ($m=2n$) obtained by averaging the $(2n)^{\text{th}}$ power of (1′), have the well-known form

$$\langle x^{2n}(t)\rangle = (2n-1)!!(\langle x^2(t)\rangle)^n ,$$

which depends on the mean-square displacement of the particle. Using the definition of the characteristic function one thus obtains, for any type of Gaussian disorder

$$p(x,t) = \frac{1}{[2\pi\langle x^2(t)\rangle]^{1/2}} \exp\left[-\frac{x^2}{2\langle x^2(t)\rangle}\right] . \tag{4}$$

The explicit form of $\langle x^2(t)\rangle$ for the correlation (2) and (3) may be readily found by averaging the square of (1′). In the following we are only interested in the limiting expressions for $t \gg \tau$ and for $t \ll \tau$, which are given by [1]

$$\langle x^2(t)\rangle = \begin{cases} \dfrac{f_0^2 t^3}{3}\left[1-\dfrac{3\tau}{2t}+\cdots\right] , & t \gg \tau & \text{(5a)} \\[2ex] \dfrac{F_0^2 t^4}{4}\left[1-\dfrac{4t}{15\tau}+\cdots\right] , & F_0^2 = \dfrac{f_0^2}{2\tau} . & \text{(5b)} \end{cases}$$

Here the particle is assumed to be initially at $x=0$, i.e.,

$$p(x,0) = \delta(x) . \tag{6}$$

In particular, in the white-nose limit ($\tau=0$) where the random force is uncorrelated [since $\lim_{\tau\to 0} h(t) = \delta(t)$], we have

$$\langle x^2(t)\rangle = \tfrac{1}{3} f_0^2 t^3 . \tag{5'}$$

While (5′) describes the familiar superdiffusive evolution [2] of a particle subjected to a white-noise force, the first term of (5b) corresponds to uniformly accelerated spreading in the presence of a static Gaussian force, which is described by the limits $\tau \to \infty$, $f_0^2/2\tau \to F_0^2$ = finite value. In the following we refer to the limit $t \gg \tau$, where the effect of the random force reduces asymptotically to that of an uncorrelated force, as the limit of quasiperfect dynamic randomness. On the other hand, the limit $t \ll \tau$, where the effect of the random force approaches that of a static force, will be termed the limit of quasistatic randomness.

Similarly the probability distribution for the velocity of the particle at any t is exactly given by [1]

$$p(v,t) = (2\pi\langle v^2(t)\rangle)^{1/2} \exp(-v^2/2\langle v^2(t)\rangle) . \tag{7}$$

Again, the simplest derivation [1] of this result consists in observing that for Gaussian randomness the mth moment $\langle v^m(t)\rangle$ of the velocity distribution, given by the average of the mth power of the first integral of (1),

$$v(t) = \textstyle\int_0^t dt' f(t') , \tag{1''}$$

reduces to

$$\langle v^m(t)\rangle = \begin{cases} 0 \quad \text{for} \quad m=2n+1 \\ (2n-1)!!(\langle v^2(t)\rangle)^n \quad \text{for} \quad m=2n , \end{cases}$$

which readily yields the Gaussian distribution (7) via the characteristic function. In the special case of the correlation (2), one finds

$$\langle v^2(t)\rangle = f_0^2(t-\tau[1-\exp(-t/\tau)]) . \tag{8}$$

Equations (4) and (7) are solutions of generalized displacement and velocity diffusion equations

$$\frac{\partial p(x,t)}{\partial t}=\frac{1}{2}\frac{d\langle x^2(t)\rangle}{dt}\frac{\partial^2 p(x,t)}{\partial x^2}, \tag{9}$$

$$\frac{\partial p(v,t)}{\partial t}=\frac{1}{2}\frac{d\langle v^2(t)\rangle}{dt}\frac{\partial^2 p(v,t)}{\partial v^2}, \tag{10}$$

which constitute our starting point for the study of first-passage times and survival probabilities in Secs. II A and II B.

A. First-passage times

For a random particle placed initially at the origin $x=0$ of a line extending from $x=-\infty$ to ∞ one is typically interested in the probability density $y(t)$ for the particle to cross either of two points $x=\pm\xi$ for the first time at time t. This probability is related to the probability $w(x,t)$ of finding the particle at a point x on the segment $-\xi\le x\le\xi$ without having crossed the edges, $x=\pm\xi$, during time t. Clearly $w(x,t)$ is given by the solution of Eq. (9) on the segment extending from $x=-\xi$ to ξ if one assumes that absorbing barriers have been erected at $x=\pm\xi$ such that $w(\pm\xi,t)=0$. This amounts indeed to discarding the points $x=\pm\xi$. The solution for $w(x,t)$ obeying this boundary condition and the initial condition (6), obtained from the method of separation of variables, is

$$w(x,t)=\frac{1}{\xi}\sum_{n=0}^{\infty}\cos(k_n x)\exp(-\tfrac{1}{2}k_n^2\langle x^2(t)\rangle), \tag{11}$$

where

$$k_n=\frac{(2n+1)\pi}{2\xi}, \quad n=0,1,2,\ldots . \tag{12}$$

It generalizes the corresponding solution for normal diffusion studied by Seshadri and Lidenberg [3]. The total probability $W(\xi,t)$ for finding the particle within $-\xi\le x\le\xi$ without ever having crossed the domains edges is then

$$W(\xi,t)=\int_{-\xi}^{\xi}dx\, w(x,t) \tag{13}$$

$$=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\exp\left[-\frac{(2n+1)^2\pi^2\langle x^2(t)\rangle}{8\xi^2}\right]. \tag{13$'$}$$

Now, by conservation of probability, the probability density for the particle to cross $x=\xi$ or $x=-\xi$ at time t is

$$y(t)=-\frac{dW(\xi,t)}{dt}, \tag{14}$$

so that the moments $T_m(\xi)$ of $y(t)$, the so-called first-passage time moments, are given by

$$T_m(\xi)=\int_0^\infty dt\, t^m y(t)$$
$$=m\int_0^\infty dt\, t^{m-1}W(\xi,t), \quad m=1,2,\ldots , \tag{15}$$

where $T_1(\xi)$ is the mean first-passage time, $T_2(\xi)$ is the mean-square first-passage time, etc.

From (13$'$) and (15), using Eqs. (5a) and (5b), we then obtain successively, for limiting cases in the random-force dominated regime:

$$T_1(\xi)=\frac{4}{3\pi}\Gamma\left[\frac{1}{3}\right]\beta\left[\frac{5}{3}\right]\left[\frac{24\xi^2}{\pi^2 f_0^2}\right]^{1/3}+\frac{\tau}{2}, \tag{16a}$$

$$T_2(\xi)=\frac{8}{3\pi}\Gamma\left[\frac{2}{3}\right]\beta\left[\frac{7}{3}\right]\left[\frac{24\xi^2}{\pi^2 f_0^2}\right]^{2/3}$$
$$+\frac{8\tau}{\pi}\Gamma\left[\frac{4}{3}\right]\beta\left[\frac{5}{3}\right]\left[\frac{3\xi^2}{\pi^2 f_0^2}\right]^{1/3}, \tag{16b}$$

for quasiperfect dynamic randomness ($t\gg\tau\to 0$) and

$$T_1(\xi)=\frac{1}{2\pi\sqrt{2}}\Gamma\left[\frac{1}{4}\right]\beta\left[\frac{3}{2}\right]\left[\frac{\xi^2}{2F_0^2\pi^2}\right]^{1/4}$$
$$+\frac{8\sqrt{2}}{15\pi^{3/2}}\beta(2)\frac{\xi}{|F_0|\tau}, \tag{17a}$$

$$T_2(\xi)=\frac{1}{4\pi^{3/2}}\beta(2)\frac{\xi}{\sqrt{2}|F_0|}$$
$$+\frac{2^{1/4}}{5\pi\tau}\Gamma\left[\frac{3}{4}\right]\beta\left[\frac{5}{2}\right]\left[\frac{8\xi}{\pi|F_0|}\right]^{3/2}, \tag{17b}$$

for quasistatic randomness ($t\ll\tau\to\infty$). Here $\Gamma(x)$ is the gamma function and

$$\beta(\alpha)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^\alpha}, \tag{18}$$

where we generalize a familiar notation [14] for sums of integer reciprocal powers of odd numbers to sums of reciprocal powers which may be fractional. One has [14] $\beta(2)=0.915\,965\ldots\equiv G$ (Catalan's constant), and the sums of fractional reciprocal powers in (16a) and (16b) and (17a) and (17b) are readily accessible on a pocket calculator if precise numerical values are required.

It may be of interest also to study the times when the velocity diffusion process defined by (8) and (10) first reaches some particular (threshold) value $|\eta|$. Thus the mean and mean-squared first-passage times to velocity values $\pm\eta$ are found to be

$$T_1(\eta)=\left[\frac{\eta}{f_0}\right]^2+\tau, \tag{19a}$$

$$T_2(\eta)=\frac{2^9}{\pi^5}\beta(5)\left[\frac{\eta}{f_0}\right]^4+2\left[\frac{\eta}{f_0}\right]^2\tau, \tag{19b}$$

for $t\gg\tau\to 0$, and

$$T_1(\eta)=\frac{4\sqrt{2}}{\pi^{3/2}}\beta(2)\frac{\eta}{|F_0|}+\frac{1}{6\tau}\left[\frac{\eta}{F_0}\right]^2, \tag{20a}$$

$$T_2(\eta)=\left[\frac{\eta}{F_0}\right]^2\left[1+\frac{16\sqrt{2}}{\pi^{7/2}}\beta(4)\frac{\eta}{|F_0|\tau}\right], \tag{20b}$$

for $t\ll\tau\to\infty$. Here [14] $\beta(4)=0.988\,94\ldots$, $\beta(5)=0.996\,15\ldots$, and we have used the values $\beta(1)=\pi/4$ and $\beta(3)=\pi^2/32$. Note that, with the replacement $f_0^2\to 2D$, the τ-independent terms in (19a) and

(19b) coincide with Eqs. (3.6) and (3.7) in Ref. [3], respectively.

It is instructive to compare the above results with characteristic (super)diffusion times associated with the moments of the distribution of displacements for a particle in a random-force field. Equations (5a) and (5b), for example, yield the following expressions for characteristic times t_1, of superdiffusion to a distance l from the origin:

$$t_1 = \begin{cases} \left[\dfrac{3l^2}{f_0^2} \right]^{1/3} + \dfrac{\tau}{6} + \cdots , & t_1 \gg \tau & (21) \\ \left[\dfrac{2l}{|F_0|} \right]^{1/2} + \dfrac{2}{15\tau} \dfrac{l}{|F_0|} + \cdots , & t_1 \ll \tau . & (22) \end{cases}$$

By comparing (16a) and (17a), respectively, with (21) and (22), one observes that the form of the scaling of T_1, with ξ and with the parameters $f_0(F_0)$ and τ is the same as the scaling of t_1, in (21) and (22). A further remarkable feature of the above results is the universal form of the leading finite τ correction in the mean first-passage times (16a) and (19a).

B. Survival probability

We now add a low concentration $c \ll 1$ of fixed absorbing traps at random positions x_i, $i=1,2,\cdots,N$, along our random chain of length $L \to \infty$. A particle placed initially at some point on the chain will move randomly until disappearing instantly when touching for the first time one of its two nearest-neighbor traps. The infinite chain is thus naturally divided up into independent adjacent segments with traps at their end points such that, on any given segment, a particle may diffuse freely until being absorbed when reaching a trap. The first step is thus to determine the probability $w_i(x,t)$ of finding the particle at point x at time t on a segment with trapping centers at the edges x_i and x_{i+1} given that it started somewhere on this segment at $t=0$. This probability is related to the probability of first passage to x_i or x_{i+1}, as discussed in Sec. II A. The total survival probability $S(t)$ is then obtained as follows, as a double average of the probability, $W_i(t)=\int_{x_i}^{x_{i+1}} dx\, w_i(x,t)$ of being anywhere on a given segment bounded by traps [9,10]. First, since at $t=0$ the random particle may be anywhere on segment i, we average $w_i(x,t)$ over a uniform initial distribution of particle positions, i.e.,

$$w_i(x,0)=\frac{1}{L} , \tag{23}$$

which normalizes to l_i/L over the ith segment of length $l_i=|x_{i+1}-x_l|$. Secondly, because of the fact that the diffusing particle may be found on any one of the above segments, of variable lengths, the survival probability reduces to the average of $W_i(t)$ for a particular segment i, over the distribution of segment lengths corresponding to randomly distributed traps. Since the traps are uniformly distributed with a concentration $c=N/L$ the probability of finding a segment of length l or, equivalently, the probability for the neighbor of a given trap to be at a distance l, is

$$f(l)=c\exp(-cl) . \tag{24}$$

In summary, for the case of particles driven solely by a random force, we first require the solution $w_i(x_1 t)$ of Eq. (9) on a segment $x_i \le x \le x_{i+1}$ of length l_i, with the boundary condition of vanishing probabilities at the end points:

$$w_i(x_{i+1},t)=w_i(x,t)=0 , \tag{25}$$

and the initial condition (23). The total survival probability, $S(t)$ is then given by [10]

$$S(t)=\sum_i W_i(t)=N\langle W_i(t)\rangle , \tag{26}$$

where $N=cL$ is the total number of traps and $\langle\ \rangle$ denotes averaging with respect to the distribution of segment lengths (24). By solving Eq. (9) by the method of separation of variables under the conditions (23) and (25) we get

$$w_i(x,t)=\frac{4}{L}\sum_{n=0}^{\infty}\frac{\sin[k_n(x-x_i)]}{k_n l_i}\exp\left[-\frac{k_n^2\langle x^2(t)\rangle}{2}\right] , \tag{27}$$

with k_n defined by (12) where l_i now replaces 2ξ. Next we use the form of $W_j(t)$ which follows from (27) to express the survival probability (26). The final result is

$$S(t)=\frac{4}{\pi^2}\int_0^\infty d\lambda \frac{\lambda}{\sinh\lambda}\exp\left[-\frac{\pi^2c^2\langle x^2(t)\rangle}{2\lambda^2}\right] , \tag{28}$$

which generalizes the result obtained earlier by Balagurov and Vaks [10] in the case of normal diffusion. In obtaining (28) the infinite series over wave numbers in (27) has been summed thanks to a change of integration variable when averaging over segment lengths. Also, one may verify that the initial condition $S(0)=1$ is indeed obeyed by (28), e.g., by using tabulated integrals [14,15].

We shall analyze (28) successively in the limits of short and of long times as defined by $a \ll 1$ and $a \gg 1$, respectively, where

$$a=\frac{\pi^2c^2}{2}\langle x^2(t)\rangle . \tag{29}$$

For each of these cases we shall distinguish between the limit of quasiperfect dynamic randomness described by (5a) with $\tau \to 0$ and the limit of quasistatic randomness defined by (5b) with $\tau \to \infty$.

1. Short-time domain ($a \ll 1$)

In this case we use the condition $S(0)=1$ to rewrite (28) as

$$S(t)=1-\frac{4}{\pi^2}\int_0^\infty d\lambda\frac{\lambda}{\sinh\lambda}\left[1-\left[\exp-\frac{a}{\lambda}\right]\right] .$$

Since for $a \ll 1$ the dominant contribution to the integral arises from values $\lambda \ll 1$, we approximate the factor

$\lambda/\sinh\lambda$ by its $\lambda=0$ limit, which yields [15] $S(t)=1-4\pi^{-3/2}\sqrt{a}$. From (5a) and (5b) we thus obtain

$$S(t)\simeq 1-2\left[\frac{2}{3\pi}\right]^{1/2} c|f_0|t^{3/2}\left[1-\frac{3\tau}{4t}\right]+\cdots , \tag{30}$$

for the limit of quasiperfect dynamic disorder, and

$$S(t)\simeq 1-\left[\frac{2}{\pi}\right]^{1/2} c|F_0|t^2\left[1-\frac{2t}{15\tau}\right]+\cdots , \tag{31}$$

for quasistatic disorder. These results for the random-force-dominated (or low-damping) regime should be compared with the slower initial decrease, proportional to $\sqrt{t}$, of $S(t)$ in the diffusion limited (or large damping) regime [10]. The latter time dependence has apparently been observed for exciton trapping in the transient regime [16].

2. Long-time domain ($a \gg 1$)

For long times such that $a \gg 1$ the contribution from the domain $\lambda \gg 1$ dominates in the integral (28). Using a saddle-point integration [saddle point at $\lambda \simeq (2a)^{1/3}$] we find

$$S(t)=16\left[\frac{a}{3\pi^3}\right]^{1/2}\exp\left[-\tfrac{3}{2}(2a)^{1/3}\right] .$$

By inserting (5a) and (5b) we then obtain successively

$$S(t)=\frac{8}{3}\left[\frac{2c^2f_0^2}{\pi}\right]^{1/2} t^{3/2}\left[1-\frac{3\tau}{4t}\right] \times\exp\left[-\frac{3}{2}\left[\frac{\pi^2c^2f_0^2}{3}\right]^{1/3} t\left[1-\frac{\tau}{6t}\right]\right] , \tag{32}$$

for quasiperfect dynamic disorder ($t \gg \tau \to 0$) and

$$S(t)=8\left[\frac{c^2F_0^2}{6\pi}\right]^{1/2} t^2\left[1-\frac{2t}{15\tau}\right] \times\exp\left[-\frac{3}{2}\left[\frac{c^2\pi^2F_0^2}{4}\right]^{1/3} t^{4/3}\left[1-\frac{4t}{45\tau}\right]\right] , \tag{33}$$

for quasistatic disorder ($t \ll \tau \to \infty$). Thus instead of the stretched exponential decrease of $S(t)$, with an exponent $-\ln S(t)\sim t^{1/3}$, in the diffusion-limited regime [10,17] we find an asymptotic exponential decrease $-\ln S(t)\sim t$ for weakly correlated dynamic randomness ($t \gg \tau$) and a shrunk-exponential decrease $-\ln S(t)\sim t^{4/3}$ for quasistatic randomness ($t \ll \tau$) in the random-force-dominated regime.

Recently Araujo *et al.* [2] have suggested the asymptotic behavior

$$S(t)\sim \exp(-c_2 t^{3/5}) , \tag{34}$$

for the case of the random-force-dominated regime with uncorrelated force fluctuations ($\tau=0$). We believe that (34) is incorrect. The reason is that individual Fourier components in the expression (27) for the probability of finding the diffusing particle on an individual segment, with absorbing traps at the edges, are proportional to $\exp[-c_1t^3/l^2]$ [Eq. (5c)] rather than to $\exp[-c_1t/l^{2/3}]$ as assumed in Eq. (2) of Ref. [2]. This is the origin of the qualitatively different forms of the asymptotic decrease in (34) and in the $\tau=0$ form of (32).

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